Katherine Moore

PBL Lesson
Algebra 2/Trig
Hoover High School
Spring 2011
Problem-Based Learning

PBL is an approach that allows students to work together in order to discover a solution to a problem that may occur in real life. Through PBL, students will gain content knowledge, team participation skills, problem-solving skills, and experience in self-directed learning. This approach increases students’ motivation and feelings of accomplishment as well. Students assume increased responsibility for their learning, while the teacher acts as a facilitator. The overall goal is to aid students in becoming life-long learners.

Students are presented with some type of “hook” that introduces them to the problem as well as gives the students a different role (such as detectives). Students are then guided through questions in order to discover what is expected of the problem. After these questions, students should begin working on the problem. The teacher should act as a facilitator and only intervene when necessary. The problem should close with a discussion of the knowledge attained through the problem.

Rationale for PBL Approach

With this problem, the PBL approach will allow students to understand real-life applications of right triangle trigonometry, especially angles of elevation and depression. The PBL approach will give students a deeper understanding of the essential unit circle. In this way, the teacher hopes that this content is embedded in the students’ minds so that there will be increased retention. Because this problem is focused on vacation, the teacher hopes that students’ motivation will increase despite “spring fever.”

Developed by: Katherine Moore   2011
Lesson Objective:
Given the opportunity to work in collaborative learning groups, Algebra 2/Trig students will practice and examine various applications of right triangle trigonometry, including angle of elevation/depression. Students will also derive the unit circle using right triangle trigonometry.

National & State Standards:
From the National Council of Teachers of Mathematics:
Use mathematical models to represent and understand quantitative relationships: identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.

From the Alabama Course of Study:
11.) Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

Pre-Instructional Activities:
The teacher will first ask students to recall the six trigonometric functions. Then, the teacher will guide students through an example problem in order to orient them to this problem.

The example problem will be:
A surveyor stands 75 feet from a building and sights the top of the building at a 65° angle of elevation. How tall is the building?

Problem Overview:
Students will become math-enthusiast vacationers. Using situations found on vacation, students will solve various problems dealing with right-triangle trigonometry. Also, students will derive the unit circle using right triangles.

Students will be given the following hook:
“Guess what? You have made it through another year of school! In order to celebrate, you are taking a trip to your favorite beach! While you’re there, you notice how much you can use right triangle trigonometry!”
Problem Analysis:
The teacher will guide students through questions in order to help students grasp the problem. The teacher will ask the following probing questions:

- What are we going to do?
- How are we going to do it?
- What time of year is it?
- What is your role?
- What mathematical ideas will we need to use?
- What will we solve for?
- Which mode should your calculator be in?
- How should we begin this process? What should we do next?

The students’ responses will be recorded in the following chart:

<table>
<thead>
<tr>
<th>Know</th>
<th>Need to Know</th>
<th>Hypotheses</th>
<th>Action Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>-Solve problems</td>
<td>-How to apply trigonometric</td>
<td>-We are math-minded vacationers.</td>
<td>-Assign groups.</td>
</tr>
<tr>
<td>-Use right triangle</td>
<td>functions</td>
<td>-We will use angles of elevation/</td>
<td>-Designate roles.</td>
</tr>
<tr>
<td>trigonometry</td>
<td></td>
<td>depression to find side and angle</td>
<td>-Work on problems.</td>
</tr>
<tr>
<td>-Use angles of</td>
<td>-Side measures of triangles</td>
<td>measures.</td>
<td></td>
</tr>
<tr>
<td>elevation/depression</td>
<td>-Angle measures of triangles</td>
<td>-We will apply sine, cosine, and tangent</td>
<td></td>
</tr>
<tr>
<td>-It’s summer!</td>
<td>-How to apply angles of elevation/</td>
<td>functions.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>depression</td>
<td>-Our calculators will be in degree mode.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-We will sketch each problem.</td>
<td></td>
</tr>
</tbody>
</table>

Group Formation:
Students will be placed in groups of 2-3 students. Each student is responsible for turning in their own work, but students may work together on the various problems. Students will assume the roles of task encourager and time keeper. If there are three students in one group, the group may decide which role will be assigned to two students. Roles include:

**Task Encourager:** The task encourager is responsible for keeping the group focused on the assignment as well as guiding conversations and activities for the successful completion of the problem.

**Time Keeper:** This student is responsible for keeping track of time and ensuring that the group manages time efficiently.
**Anticipated Learning Issues:**
- Recall and apply trigonometric functions (sine, cosine, tangent)
- Solve problems using angle of elevation/depression
- Deriving the unit circle

**Group Deliberation & Problem-Solving Activities:**
Students will have 2 days to complete this project. On the first day, students will be introduced to the problem, and they will work on the various problems. If time permits, they can start on Part 2, deriving the unit circle.

On the second day, students will complete Parts 2 and 3, where they will derive the unit circle. The teacher will provide help in understanding the directions of Parts 2 and 3. In Part 3, the teacher will call the class’ attention to reflecting triangles across the axes. This lesson will conclude with a discussion.

**Formative Evaluation:**
Students will be completing a small packet during this problem. This packet will be collected and graded using a criteria sheet. This criteria sheet also assesses working in a group: staying on task, contributing ideas, and respecting group members.

**Problem Follow-Up:**
During the class discussion, some students may show the class their unit circle using the document camera. The teacher will ask the following (and any additional) questions:

  - What was the most difficult part of the problem?
  - What is the importance of right triangle trigonometry? Why do we learn it?
  - If you could do this problem again, what would you do differently?

The teacher will also review the problem analysis chart completed at the beginning of the lesson, allowing students to modify the chart with their newfound knowledge.

<table>
<thead>
<tr>
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<th>Hypotheses</th>
<th>Action Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-Unit circle will be useful later.</td>
<td>-Find triangle measures</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-Cut out triangles</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-Plot points on axes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-Create unit circle</td>
</tr>
</tbody>
</table>
Correctives & Extensions:
Correctives: The following activities are meant to aid students who fail to display an acceptable mastery of the content during the previous lesson.
   Students who fail to master the content will be allowed to go outside and have hands-on experience with angle of elevation/depression.

Extensions: The following activities are provided to aid students who excel and achieve mastery to a level that requires additional material to challenge students.
   Students who achieve at a high level of mastery will be required to research radians. They will then convert the angle measures on their unit circle to radians. This will prepare them for learning how to use the unit circle later in this class as well as in future math classes.

Resources:
Teacher: textbook, packets, calculator with basic trigonometric functions, criteria sheet
Student: textbook, packets, calculator with basic trigonometric functions, criteria sheet, scissors

Developed by: Katherine Moore    2011
It’s Finally Vacation!

Guess what? You have made it through another year of school! In order to celebrate, you are taking a trip to your favorite beach! While you’re there, you notice how much you can use right triangle trigonometry!

Part One:

Directions: Sketch each scenario. Solve the various problems using right triangle trigonometry (including angle of elevation and depression. SHOW YOUR WORK!

1. You just arrived at your favorite beach destination. As you leave the airport, you notice a plane in the sky at an angle of 60°. The plane is directly above a sign that is 6500 ft. from where you are standing. How high is the plane flying?

2. On your first adventure, you decide to climb to the top of a cliff overlooking the ocean. When you arrive at the top, you spot a sailboat in the water at an angle of 45°. The cliff you just climbed is 100 feet high. How far away is the sailboat from the base of the cliff?

3. As you start on your journey back down to the bottom, you see a huge tree. You decide that you must know the height of the tree before you can continue. Luckily, it’s a sunny day, and you observe that the tree’s shadow is 25 ft. long. You stand at the end of the shadow, looking up at the top of the tree at an angle of 35°. How tall is the tree?
4. As you continue down from the cliff, you stumble upon a beautiful garden outlined with timbers that just happen to form a right triangle! You have a measuring tape, and you see that the hypotenuse is 7 feet, and one angle is 20°. You really don’t want to measure all of the other lengths and angles, so you decide to use trigonometry! Find the missing side lengths and angle measures.

5. The next day, you decide to climb to the top of the lighthouse. After climbing 150 feet, you spot your friends down below at an angle of 30°. How far away are your friends from the bottom of the lighthouse?

6. Now, your friends walk towards the lighthouse so that you must look down at an angle of 40°. How far away are your friends from the bottom of the lighthouse?

7. How many feet did your friends walk towards the lighthouse?
8. After the lighthouse, you and your friends decide to go shopping. As you’re shopping, you spot a second-story window with just what you’re looking for. You are 50 feet from the base of the building, looking up at the window at an angle of 13°. If you could draw an imaginary line from yourself to the window, how long would it be?

9. Using trigonometry, how high is the window?

10. After leaving the shop, you notice a street art show. There is one piece of artwork that catches your eye; of course, it’s in the shape of a right triangle! You decide to measure it to see if it will fit on your wall at home. One angle of the triangle is 50°. The side opposite of this angle measures 3 feet. Find the missing side lengths and angle measures.

11. As you soak up the sun on the beach, you decide to draw your own right triangle in the sand. If the two sides (not hypotenuse) are 10 inches and 12 inches, respectively, find the length of the hypotenuse and angle measures.
Beach Day Fun!

You’re bored on the beach, so you decide to give your brain some mental exercises. What better way to spend your vacation!

Part Two:

Using the Equilateral Triangle:

1. In the equilateral triangle, label the angle measures and allow the side length to be 1.

2. Now, bisect the triangle in order to get two right triangles.

3. Label the bisected angle and side.

4. Find the length of the bisector. (Hint: There’s a helpful formula you should know from Geometry involving right triangles…)

5. Label the length.

6. What are the sine, cosine, and tangent for each of the angles (30°, 60°, 90°) in your right triangle? Make sure to rationalize the denominator if needed.
**Using the Square:**

1. Label the sides of the square to have length 1.

2. Bisect the square in order to make two right triangles.

3. Label the angle measures.

4. Find the length of the hypotenuse of the new triangles.

5. Label the length.

6. Now, we want the hypotenuse to equal length 1. How can we do this? What are the new side measures?

7. What are the sine, cosine, and tangent for each of the angles (45°, 45°, 90°) in your right triangle? Make sure to rationalize the denominator if needed.
Part Three:

1. Cut out the attached triangles (for Part 3).

2. Using the axes below, start with your triangle with angle 30°. Trace its hypotenuse onto the graph, and plot a point at the end of the hypotenuse.

3. Continue this process with angles 45° and 60°.

4. In order to complete the second quadrant, you will reflect the first quadrant over the y-axis.

5. Follow the same steps to complete the second quadrant.

6. Next, reflect the second quadrant over the x-axis in order to complete the third quadrant.

7. Finally, reflect the third quadrant over the y-axis in order to complete the fourth quadrant.

8. Connect the dots in order to form a circle.

9. At each point, label the point using the following form: (cosθ, sinθ)

10. Congratulations! You have derived the unit circle! You did all this on the beach! Wow! Great job!
The Unit Circle:
**Use for Part Two:**
Label these shapes.

![Triangle](triangle.png)  
![Square](square.png)

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**Use for Part Three:**
Cut out these triangles.

30° - 60° - 90°  
45° - 45° - 90°  

![30-60-90 Triangle](triangle306090.png)  
![45-45-90 Triangle](triangle454590.png)

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Criteria Sheet:

Part One:

Sketch of each problem _____ / 7 pts.

Shows comprehension and provides correct answer (2 pts. each) _____ / 22 pts.

Part Two:

Shows completion and accuracy of the following: _____ / 15 pts.

Equilateral Triangle (7 pts.), Problem 4 (2 pts.), Problem 6 (6 pts.)

Part Three:

Shows completion and accuracy of the following: _____ / 15 pts.

Square (7 pts.), Problem 4 (2 pts.), Problem 6 (3 pts.), Problem 7 (3 pts.)

Group Work: _____ / 6 pts.

Rating Scale: 2 = great 1 = average 0 = needs improvement

- Staying on task 2 1 0
- Contributing ideas to group 2 1 0
- Respecting group members 2 1 0

Total: ________ /65 pts.